

Remarks on emergence of interactions and space-time

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Abstract

In a previous paper the author has shown that the electromagnetic interaction can be understood as a consequence of the quantum mechanical rule, that the kinematics of isolated two-particle systems are described by irreducible two-particle representations of the Poincaré group. In the present paper further consequences of this rule with special attention to classical and quantum gravity and to the emergence of space-time are discussed.

Keywords: conformal gravity; quantum gravity; emergence of space-time.

1 Introduction

This paper is based on the assumption of a fundamental Poincaré invariance of nature. The origin of this invariance is not subject of this paper. Basic rules of quantum mechanics then require that not only elementary particles, but also isolated systems of elementary particles are described by irreducible representations of the Poincaré group.

Irreducible representations of the Poincaré group can be labelled by the (constant) values of two Casimir operators P^2 and W^2 ,

$$P^2 = P_\mu P^\mu, \tag{1}$$

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where P_μ is the 4-momentum of the system, and

$$W^2 = -W^\mu W_\mu, \text{ with } W_\sigma = \frac{1}{2} \epsilon_{\sigma\mu\nu\lambda} M^{\mu\nu} P^\lambda, \quad (2)$$

refers to the angular momentum of the system.

In a previous paper [1] consequences resulting from (1) for a two-electron system were discussed. In this case the total 4-momentum is given by

$$P_\mu = p_1^\mu + p_2^\mu. \quad (3)$$

It is obvious that the two-particle mass-shell relation (1) with (3) leads to a correlation between the individual momenta of the electrons. The author has shown in [1] that the quantum mechanical formulation of this correlation has the structure of the electromagnetic interaction, as described by the perturbation algorithm of quantum electrodynamics. The coupling constant, derived from geometrical properties of this correlation, was found to be equal to the empirical value of the electromagnetic fine-structure constant.

In the present paper we will, without restriction to electrons, discuss consequences of the constancy of the second Casimir operator (2). It concerns the *angular momentum* generated by the *relative motion* of two elementary particles, described by an irreducible representation of the Poincaré group.

Quantum mechanically, the relative motion is described by a wave function, formed by spherical functions, corresponding to the same value of W^2 . In the classical (large N) limit the spherical symmetry property of these functions *does not fade away, but is preserved*. In the large N limit spherical functions describe orbits with a circular symmetry (cf. e.g. [2]). Therefore, two particles in an irreducible two-particle representation typically are *not* in straight uniform motion, as classical independent particles would be.¹ (I hope, Sir Isaac will forgive me my heretical thoughts.) They rather behave like two particles being forced into a circular orbit by a kind of attractive “gravitational force”.

In the following the geometrical properties of this force, which obviously is the outcome of the quantum mechanical description, will be analysed in

¹ Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare. Isaac Newton: *Philosophiæ Naturalis Principia Mathematica*.

more detail. It will be shown that gravitational forces, like electromagnetic forces (cf. [1]), emerge from the quantum mechanical nature of matter.

2 Parameter space-time vs. physical space-time

Given an elementary particle, described by an irreducible representation of the Poincaré group in a state space with eigenstates $|p\rangle$ of the 4-momentum p , then states “in space-time” can be defined by superposition of 4-momentum eigenstates

$$|\mathbf{x}, t\rangle = (2\pi\hbar)^{-\frac{3}{2}} \int \frac{d^3\mathbf{p}}{|p_0|^{\frac{1}{2}}} e^{-\frac{ipx}{\hbar}} |p\rangle, \quad (4)$$

with parameters $x = (\mathbf{x}, t)$. The parameters x form a space with the same metric as the energy-momentum space. States (4) are “localized” at the position \mathbf{x} . So we can say that the parameter space has also a “physical” meaning in the sense that it is a space, where particles can physically be placed.

Notice that (4) contains a *definition* of space-time on the basis of momentum eigenstates. Based on (4) we can define a *position operator* by

$$X_\mu = -i\hbar \frac{\partial}{\partial p^\mu}. \quad (5)$$

Space-time, thereby, is *derived* as a property of matter. There is no need to assume a prior existence of space-time as an independent entity.

The relation between space-time and energy-momentum contains the Planck constant \hbar . This is the result of independent scales for x and p . We can avoid this constant by replacing p by the *wave vector* k , defined by $p = \hbar k$, which in this context is a more natural choice.

Now consider *two* elementary particles, described by an irreducible two-particle representation of the Poincaré group. Because of the constraints by the two-particle mass shell relation (1) with (3), it is not possible to simultaneously construct localized states (4) for both particles. Therefore, when two or more particles are considered, the physical property of x -space is somewhat lost, but it still can serve as a useful parameter space for *wave*

functions. So we have to be careful, not to mix up *parameter* space-time with *physical* space-time. *Physical* space-time will in the following be understood in the sense of the *expectation value* of operator (5).

As a pure mathematical construct, parameter space-time is not limited by any “physical” scale, such as the Planck length. So it does not make sense to try its “quantization” at Planck scales, in the hope to find an access to “quantum gravity”. On the other hand, *physical* space-time is quantized right from the beginning, as it is defined by expectation values of the position operator. This means that the classical concept of space-time breaks down at scales, where quantum effects become noticeable, and this happens not at Planck scales, but already at atomic scales.

An incorrect mix-up of parameter space-time and physical space-time becomes obvious in the wide-spread opinion that a theory of quantum gravity is so difficult, because quantum mechanics is defined on space-time, while in quantum gravity this very space-time continuum must be quantized.

In contrast to parameter space-time, physical space-time has a natural scale. Such a scale is e.g. given by the “Bohr radius”

$$\frac{\hbar}{c m_e \alpha} \tag{6}$$

of the electron in a hydrogen atom. This scale is determined by electromagnetic interaction, which in [1] has been shown to be a property of irreducible representations of the Poincaré group, and by the electron mass. So this mass takes over the role of the (hypothetical) Planck mass in characterizing a “smallest length”.

3 Geometry of physical space-time

As mentioned in the introduction, within an irreducible two-particle representation, the motion of the particles relative to each other is determined by a well-defined angular momentum. The associated Casimir operator (2) is a constant of motion. Quantum mechanics describes this angular momentum in (parameter) space-time by spherical functions, which in the large N limit lead to spherical probability distributions with the shape of circular orbits. Since there is no external force to keep the particles on these orbits, we are

led to the alternative interpretation that, within irreducible representations, physical space-time - in contrast to parameter space-time - in general has a curved metric.

The curvature is *not* obtained by an active deformation of a given space-time continuum, but by the *ab initio* construction of space-time from (an entangled superposition of) momentum eigenstates within an irreducible two-particle representation. Viewed in this way, it appears more or less trivial that the distribution of energy-momentum in space-time is reflected in the metric of physical space-time, and it would be surprising, if it were not.

The connection between energy-momentum space and space-time is given by the factor $e^{-ipx/\hbar}$ in (4). This factor is invariant under two simultaneous conformal transformations

$$x \rightarrow \lambda^{-1}x \tag{7}$$

and

$$p \rightarrow \lambda \ p . \tag{8}$$

By these transformations, not only parameter space-time, but also physical space-time is subjected to a scaling, which changes any probability distribution in space-time by a scaling factor λ^{-1} , but keeps the form of this distribution invariant. The symmetry, defined by these transformations, means that the linear size of a structure in space-time is inversely proportional to the magnitude of energy-momentum that defines this structure via connection (4) - accordingly, a *curvature*, associated with this structure, is directly *proportional* to energy-momentum.

Now consider a two-particle system, formed by a first (heavy) point mass and a second (light) point mass, acting as a test particle. The system shall quantum mechanically be described by an irreducible two-particle representation of the Poincaré group. In the classical limit the relative motion of the particles then forms an orbit with a circular symmetry. Following Newton's first law, we can understand this orbit as the result of a force that acts perpendicular to the velocity vectors of the particles. This force generates a linear momentum perpendicular to their actual velocities. Such a momentum is described by the momentum flux $T^{ik}, i \neq k$ ($i, k = 1, 2, 3$) of the energy-momentum tensor $T^{\mu\nu}$. Diagonal elements of $T^{\mu\nu}$, obviously, do not

contribute to the centripetal force. Although, for reasons of general covariance, we have to allow for components of $T^{\mu\nu}$ that are obtained from T^{ik} by Lorentz transformations, the trace of $T^{\mu\nu}$ is invariant under these transformations. Therefore, the deviation of the kinematics of the first particle from a straight uniform motion can, in principle, be described by the *traceless part of the energy momentum tensor*. Because the total linear momentum is conserved, the test particle must contribute an opposed flux of linear momentum. Metaphorically speaking, both particles exchange momentum.

With this in mind, we now try to express the centripetal forces by a non-Euclidean metric of space-time. Consequently, we have to look for a relation between the curvature of the orbit and the traceless part of the energy-momentum tensor, as “causation” of the curvature. (Einstein gravity, with the goal to replace Newton gravity, consequentially uses the trace of the energy-momentum tensor instead. Both approaches are in a sense complementary, as far as spherically symmetric solutions are concerned.) According to what was said above about conformal scaling, the curvature must be proportional to the scaling of momentum. Therefore, the curvature experienced by the test particle must be *proportional to the traceless part of the energy-momentum tensor* of the first particle.

A curvature tensor that can be set proportional to a traceless energy-momentum tensor must itself be traceless too. Such a tensor is the Weyl tensor $C^{\mu\nu\sigma\tau}$, which is the traceless part of the Riemann curvature tensor $R^{\mu\nu\sigma\tau}$. From the Weyl tensor a traceless gravitation tensor $W^{\mu\nu}$ can be derived [3]. This tensor can then be set into relation to the traceless part of the energy-momentum tensor $T^{\mu\nu}$. Examples of traceless energy-momentum tensors, based on different models of matter, can be found in [3]. Here we can simply subtract the trace from the energy-momentum tensor to make it traceless. This leads to the field equations of *conformal gravity*

$$W^{\mu\nu} = G_{conf} \left(T^{\mu\nu} - \frac{1}{2} T^\alpha_\alpha g^{\mu\nu} \right) \quad (9)$$

with a “gravitational constant” G_{conf} .

In order to get a feeling for the strength of the interaction described by (9), let us in $T^{\mu\nu}$ replace momenta by wave vectors. The relation between curvature and wave vectors, as defined by (4), then does not include any

“physical constant”, which means that the coupling constant in (9) becomes dimensionless. G_{conf} will then consist only of geometric factors, e.g. some powers of π , resulting from setting a linear momentum in relation to a spherical geometry. In comparison to Newton-Einstein gravity, where the “sources” are masses, wave vectors represent much “weaker” sources, as long as low-energy processes are concerned. A measure of their weakness is the relation between wave vector (= inverse of de Broglie wave length) and inverse of the Compton wave length. The Compton wave length, associated with the electron mass, has a value of 2.426×10^{-10} cm, whereas the unit for numerical values on the right hand side of (9) is the unit wave vector (with the value of 1). This results in a relation of 2.426×10^{-10} . Compare this with Newton-Einstein gravity, where the weakness of gravitational forces is expressed by a small gravitational constant with the value (in cgs-units) of 6.7×10^{-8} .

Conformal gravity has gained interest in recent years, because it may solve the problems associated with “dark matter” as well as with “dark energy” [3, 5] without additional *ad hoc* assumptions. The notorious problem of “ghosts” seems to have found a solution in recent papers [4, 6]. Within scales of our solar system, conformal gravity is known to deliver the same results as Einstein’s theory of general relativity, which is based on the Riemann curvature tensor, rather than on the Weyl tensor [5].

It is obvious that the equations (9) describes a *classical* theory of gravitation. Then what is its quantum mechanical analogue? Since we just have sketched a connection between quantum theory and classical conformal gravity, we are able to give an answer to this question: The quantum mechanical basis is nothing other than an irreducible representation of the Poincaré group. Or, in other words, *there is no “quantum gravity”* at all, except for the common rules of relativistic quantum mechanics. The situation is the same as in quantum electrodynamics, as discussed in [1]: Interaction *emerges* from the restrictions of the two-particle state space, imposed on by the requirement of irreducibility.

4 Conclusions

Reasons have been given as to why both electromagnetic and gravitational interactions can be understood as basic properties of relativistic quantum mechanics, more precisely, as properties of irreducible two-particle representations of the Poincaré group. In a nutshell, these interactions are quantum mechanical properties of matter. There are no interactions at all, if interactions are understood in the sense of being provided by coupling to *exterior* bosonic fields. There are rather correlations *within* the quantum mechanical state space of matter, as a result of irreducibility of two-particle representations. These correlations lead to quantum electrodynamics and to the equations of classical conformal gravity.

Physical space-time then turns out as just another quantum mechanical property of matter. Its geometry in the large is determined by the equations of conformal gravity. Its scale in the small is defined by electromagnetic interaction and by masses of particles involved in this interaction. Electromagnetic (and gravitational) interactions provide the basis for building extended atoms, molecules and macroscopic bodies to fill up space-time. Electromagnetic interaction provides photons, which can be used to unveil the geometry of space-time to an observer. Needless to say, electromagnetic interaction establishes a causal structure in space-time. It is these interactions that make the difference between a parameter space and physical space-time. Therefore, the emergence of physical space-time goes in parallel with the emergence of interactions.

The validity of classical space-time ends at scales, where quantum mechanics becomes effective. These scales are related to the electron mass rather than to the Planck mass. For the latter there is no room left, because it is not possible to construct a mass from \hbar , c and G_{conf} .

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